

Automatic Error Correction Using P3 Response Verification for a Brain-Computer Interface

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Abstract

Recent advances in signal processing have made brain-computer interfaces (BCIs) feasible for use as an alternative control option for those with severe motor impairments. It is possible to increase the accuracy of a BCI system by requiring users to make multiple choices to control a single item, but this comes at a cost of reduced system speed. We present experimental results from an evoked potential BCI used to control items in a virtual apartment and show the existence of the evoked potential P3 component in responses to successfully controlled goal items. A reduced response exists when items are accidentally controlled. The presence of the P3 component in responses to goal items means that it can be used for automatic error correction. Off-line experiments have been run and with a theoretical mean improvement in recognition from 78% to 85%, we show a statistically significant improvement ($p < 0.004$, Wilcoxon test, two-tailed) in accuracy of 3% using the variable averaging algorithm. Since the error correction technique depends on responses to controlled items in a particular experimental paradigm rather than the control of the items themselves, it may be possible to use this response in non-evoked potential BCI systems. Future work is discussed in this context.

1 Introduction

Recent research has shown brain-computer interfaces to be an alternative control method for those with severe motor impairments caused by diseases such as Amyotrophic Lateral Sclerosis (ALS) as well as by spinal cord injuries (Wolpaw et. al., 2002),(Perelmouter et. al., 2000),(Birch and Mason, 2000), (Obermaier et. al., 2001), (Kennedy et. al., 2000). Even with this work, BCIs remain slow and prone to error with system bit rates ranging from 6 bits/minute in the Thought Translation Device as used by actual patients (Birbaumber et. al., 1999) to much higher speeds of around 85 bits/minute for an off-line experiment with an evoked potential-based BCI (Meinicke et. al., 2003). Given that the bit rate of a typist who can type around 60 words/minute is approximately 1200 bits/minute, there is much room for improvement.

There are two main ways that the bit rate of a BCI may be improved: increase the speed at which a BCI operates or increase the accuracy of the BCI system. The speed of the BCI may be changed by making clever interfaces, by increasing the speed of making a control decision, or even by predicting what decision the user wants to make. Ways to use BCIs for practical pointing devices are now being studied (Mason et. al., 2003). Increasing the accuracy of the BCI system is another possibility and has been looked at the most heavily.

Many algorithms have been looked at to increase the classification rate of both evoked potential-based BCIs and spontaneous EEG BCIs. These methods include simple linear methods such as averaging EEG signals over multiple trials of the same stimulus (Farwell and Donchin, 1988) as well as correlations between individual trials and predetermined subject averages (Bayliss 2000). Machine learning algorithms have been used to improve BCI accuracy with techniques such as Independent Component Analysis for the reduction of artifacts (Jung et. al, 1998) and support vector machines for increasing overall accuracy (Meinicke et. al., 2003). Combinations of algorithms are being studied and it is hoped that signal classification will continue to improve.

There is another way to increase the accuracy of a BCI system. We can make use of extra information embedded in the signal for making decisions. This possibility has not been heavily studied, since increasing accuracy by using more information may actually decrease the speed of a system and the end bit rate. Combining features such as the

movement-related potential and event-related desynchronization combination proposed by Dornhege et. al. are preferable when they do not decrease the end bit rate (Dornhege et. al, 2003).

Wolpaw et. al. have suggested the use of response verification (RV) for applications that need a high accuracy more than they do a high bit rate (Wolpaw et. al., 1998). Response verification occurs when the classification of a choice for an individual has been made and the individual is then asked to verify that choice by making a further choice. Two types of errors are possible in a BCI - missing the correct choice (a false negative error) or making a choice accidentally (a false positive error). Response verification may improve accuracy by reducing the occurrence of false positive mistakes in the data. Experimentally, response verification has led to higher accuracies at the cost of a lower overall bit rate due to the requirement of having the user make an extra choice.

We propose to use automatic error correction to increase accuracy without decreasing the bit rate of a BCI system. We are not the first to suggest automatically correcting errors. Parra et. al. perform automatic response error correction for users accidentally hitting a mouse button in a visual discrimination task (Parra et. al., 2003). They use the error-related negativity (ERN) event-related potential for automatic correction. This potential is known to occur when subjects mistakenly respond to a stimulus and realize that they've made a mistake. Unfortunately, the ERN signal is difficult to use in a BCI since subjects do not normally "accidentally" make a mistake - instead a combination of faulty signal classification and faulty subject control over EEG signals combine to cause mistakes.

Since the error related negativity is difficult to use in a BCI, we propose an error correction method that depends on the use of the evoked potential P3 component. The P3 component of the evoked potential was independently reported by Chapman and Bragdon (1964) and by Sutton et. al. (1965). This component is a positive wave peaking at around 300 ms after task-relevant stimuli. While the P3 component is evoked by many types of paradigms, the most common factors that influence it are the frequency of stimulus occurrence (less frequent stimuli produce a larger response) and task relevance. Figure 1 shows the presence and absence of this component.

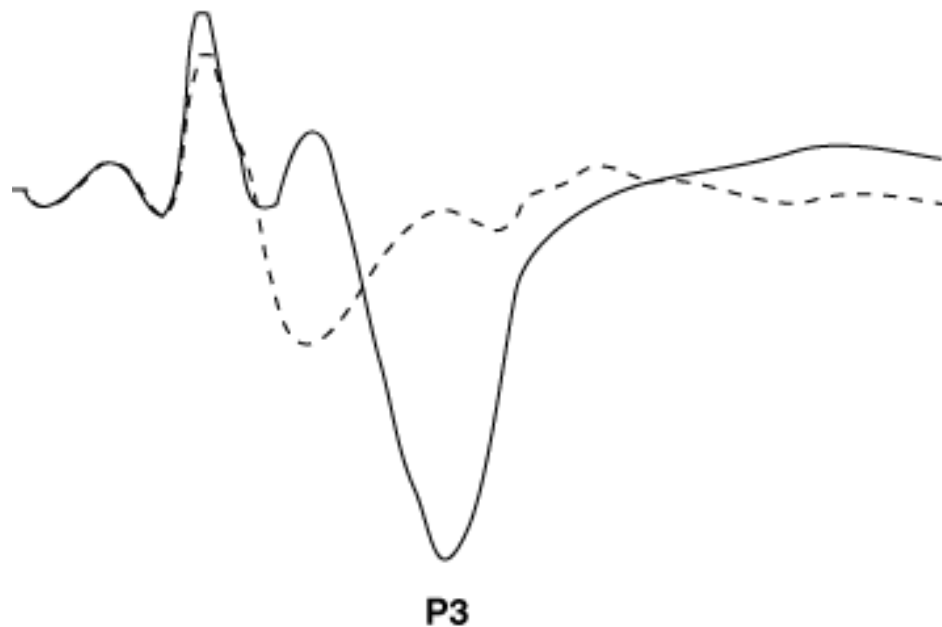


Figure 1: The solid line captures the general P3 component response average over many trials. The dashed line represents an average where the stimulus does not evoke a P3 component.

After off-line analyses of BCI data collected for controlling items in a virtual apartment, it was discovered that the P3 component exists after a goal item has been controlled and a reduced response occurs after a control mistake. Mistakes occur when an item is triggered and the subject did not mean for it to be triggered. The difference between the P3 component for goal item responses and mistakes may be used for automatic error correction and it will increase the bit rate of the system as long as the majority of corrections correct mistakes rather than cancel intended subject goal control.

Off-line experimental results indicate that with a theoretical possible increase in accuracy from 78% to 85%, automatic response verification can increase the accuracy to maximum of 81%. This improvement is shown using two simple and well-understood algorithms: peak picking and correlation. The simplicity of the algorithms and resulting improvements show the robustness of the effect. It is likely that more complicated machine learning algorithms may improve classification results.

Since automatic error correction is based on the evoked potential response after controlling an item and not on the signals used for triggering the control of an item, the possibility exists for using this technique in multiple types of BCIs. We discuss future work in this context.

2 The Virtual Reality Apartment Experiment

The Virtual Reality (VR) apartment experiment compared the similarities and differences between the evoked potential P3 component in virtual reality (VR) and looking at a computer monitor while using an on-line P3 signal-based BCI. Results show that the P3 signal is robust between different environments, although the system bit rate was fairly low at 13.24 bits/minute (Bayliss, 2003). Off-line analysis shows that an evoked potential P3 component exists in evoked potential responses after selecting an item while a reduced signal exists for false positive control mistakes.

2.1 Experimental Set-up

The experimental set-ups for the VR and non-VR environments were almost identical. The main difference was that all graphics for the non-VR environment were rendered on a 21" Silicon Graphics monitor whereas the VR environment conditions were displayed in a head-mounted display (HMD).

Five objects could be controlled by the user in the virtual apartment as shown in Figure 2. These items consisted of a lamp, stereo system, television set, a Hi command, and a Bye command. The lamp, stereo, and television all worked as toggle switches to turn the items on/off. The Hi and Bye commands made a three-dimensional graphics figure appear (for Hi) or disappear (for Bye). All responses to commands were visual - for instance musical notes appeared over the stereo when the stereo was on.

A sphere associated with each controllable object blinked in the environment; when visible, it had a semi-transparent red coloring. Semi-transparency was used so that blinking spheres would be less distracting to subjects concentrating on one specific sphere for a task. Approximately once per second, a stimulus was provided when the sphere on a randomly chosen item appeared. The stimulus would last for approximately 250ms. The stimulus presentation rate varied by up to 16 milliseconds in a random manner. The P3 evoked potential response occurs for task-relevant stimuli. To make the red sphere flashes on the controllable object task-relevant, subjects had to count the flashes on a goal item.

Seven electrode sites were arranged on the heads of nine subjects with a linked mastoid reference. Sites FZ, CZ, PZ, P3, P4, as well as an upper and lower vertical electro-oculographic (EOG) channel were used from the International 10-20 system of placement (Jasper, 1958). For on-line recognition and analysis, EOG artifacts were regressed out of the signals of interest using the algorithm by Semlitsch (Semlitsch, 1986). The EEG signals were amplified using Grass amplifiers with an analog bandwidth from 0.1 to 100 Hz. Electrode impedances were between 2 and 10 kOhms for all subjects. An epoch size from -100 ms (prior to stimulus onset) to 1500 milliseconds was specified for a total epoch size of 1600 milliseconds. The data were recorded continuously and saved to a file.

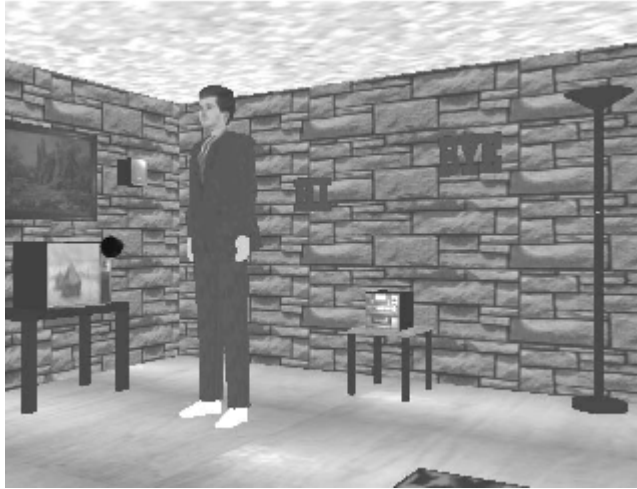


Figure 2: A sample scene from the virtual apartment. The television, stereo, HI sign, BYE sign, and lamp are all controllable items. In this scene, a red sphere on the television set is blinking.

2.2 The Experiment

The experiment consisted of four tasks:

- Calibration: The subject counted the number of sphere flashes located on the virtual lamp on a monitor.
- VR: The subject was fully immersed in the virtual apartment while wearing a HMD.
- Monitor: The subject looked at the virtual apartment on a monitor.
- Fixed Display: The subject looked at the virtual apartment on a fixed screen inside of the HMD. This represented a condition between truly immersive VR and watching a computer monitor, since it took place inside of the HMD, but had a fixed screen like a computer monitor.

The Calibration task was used to train a signal classification algorithm on a particular subject's P3 component response. A total of 300 stimulus presentations were presented to each subject. In this task, subjects were told to count only the lamp sphere flashes; thus, in this task only the lamp sphere flashes were task-relevant and these flashes should have caused a P3 component response. Since the spheres flashed randomly over the five controllable items, 60 ± 5 lamp flashes occurred over the course of five minutes.

Tasks 2-4 were accomplished in a randomized block order and lasted for approximately 5 minutes (250 stimulus presentations with the sphere flashed randomly on items). These tasks involved on-line single trial classification of the P3 component in order to control the different items in the apartment. The time taken for these trials depended on how many items were controlled, as subjects received feedback for each item for which the signal classification algorithm classified the trial as a P3 trial.

Due to the difficulty of signal classification, false positive mistakes (accidental control) were possible as well as true goal control. In tasks 2-4, the subjects received English instructions at the bottom of the screen indicating what goal to achieve and each subject attempted to achieve that goal by counting the number of flashes on the sphere located on that particular goal item. During each task, the goal was chosen randomly and the subject tried to achieve the goal for up to 50 presentations of the goal stimulus. When the goal was achieved, an action involving visual feedback occurred in the virtual apartment (for example, the room was lightened when the light was turned on). During the waiting period for this visual feedback (1.5 seconds), no new stimuli were presented. Then, the next goal was randomly chosen.

While the experiment involved on-line classification and feedback, an off-line analysis was done to compare the obtained P3 component signals between different conditions. Only epochs with a maximum vertical EOG signal of less than 50 microvolts were used. This reduced the possibility of EOG contamination of the averages. From this analysis it was discovered that the P3 signals obtained in the different environments were not significantly different from each other. Figure 2 shows this result.

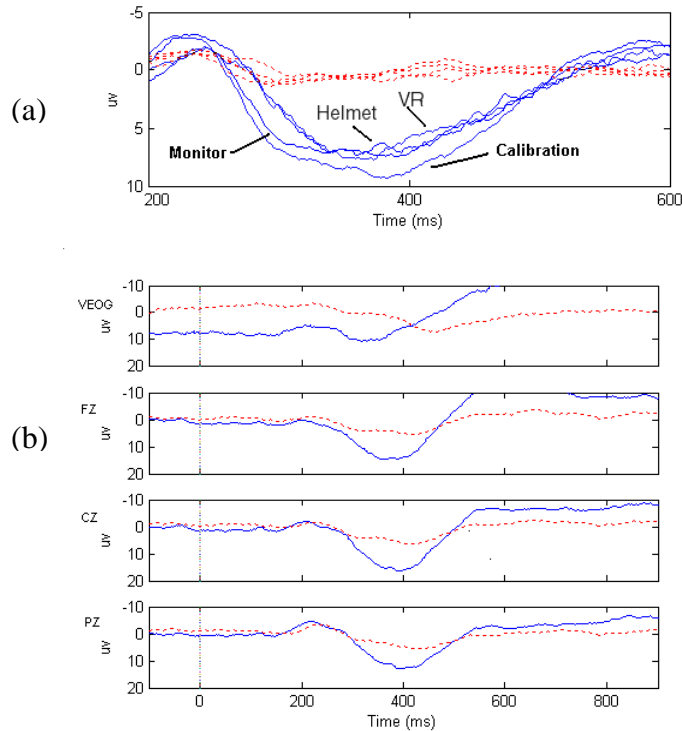


Figure 2: (a) The grand averages used to trigger items (goals) at site PZ (solid lines) shown with the grand averages for non-goals (dashed lines) in each experimental condition. (b) The grand averages over the nine subjects for responses after goal item control (the solid lines) and mistakes in item control (the dashed lines).

2.3 The appearance of the evoked potential P3 component

Since all items are controlled in a discrete manner, evoked responses where items are successfully controlled may be examined. The P3 component occurs when subjects choose a goal item and a reduced signal occurs when a mistake in classification is made. This makes sense when one considers that the triggered control item is task relevant itself. Grand averages showing this result appear in Figure 2. Since subjects could have blinked or moved directly after an item was controlled, all trials used in the grand averages had a maximum recorded vertical eye movement of less than 50 microvolts. The grand average for vertical eye movement is shown and while it is not flat, there are no peaks around 400ms when the maximal P3 component appears.

The maximum signal for FZ and CZ are slightly larger than the signal for PZ. It is possible that subjects found controlling items in a virtual apartment to be “novel” and that would lead to a more frontal response. Controlled goal items are task relevant because the subject achieves control and may go on to the next task. It is hypothesized that false positive mistakes do not generally cause this response since subjects did not have to correct errors in this experiment, so they possibly ignored them.

3 Methods

3.1 Using an evoked-potential for error correction

The method to use an evoked potential signal for response verification (RV) is similar to the RV procedure described in Wolpaw et. al. (1998). However, instead of prompting the user to verify the selection with a Yes/No trial, the system automatically keeps the selected item only if the evoked potential component is present. If p is the single-trial accuracy for selecting goals and rejecting non-goals, and q is the accuracy for the goals kept when selected non-goals are discarded during response verification, then the predicted RV proportion decided, D , is:

$$D = pq + (1 - p)(1 - q) \quad (1)$$

This is the sum of the probability of keeping a selected goal (correct trial) and keeping a selected non-goal (incorrect trial). The predicted RV accuracy, C , is the probability a decision is correct and may be represented as:

$$C = \frac{pq}{pq + (1 - p)(1 - q)} \quad (2)$$

When prompting the user to verify the selections, the user's q probability is very similar to the user's single-trial accuracy, but in automatic response verification the user is not actively responding. This makes it difficult to predict the q probability. In the current experiment, the q was found to be less than the user's single-trial accuracy, with a mean of 59% ($p < 0.01$).

3.2 Off-line Analysis

The epochs corresponding to the nine subjects' previously recorded responses after triggering items from the virtual apartment experiment Monitor condition were labeled as goal and non-goal responses. The Monitor condition was chosen as the most common type of interface that users use with real systems. If a goal response consisted of the lamp being lit when it was the goal, a non-goal response would be the TV turning on when the lamp was the goal. These were further divided into responses with classified P3 components and components where the P3 signal was determined to be absent for each algorithm used in classification. This yielded four categories of response: goal with P3 present, goal with P3 absent, non-goal with P3 present, and non-goal with P3 absent.

Peak picking and correlation were the two algorithms used for classification of the evoked potential P3 component in the FZ, CZ, and PZ channels. Vertical electro-oculographic (VEOG) artifacts were regressed out of the signals of interest using the algorithm by Semlitsch (1986). Response epochs containing signal amplitude greater than 90 microvolts were ignored. This allowed for noisy response trials not to be counted in the analysis. Any ignored trials led to items remaining triggered. For example, if the lamp is lit, it would stay lit. If the P3 component is classified on FZ, CZ, or PZ then the response would be kept. This technique was used for both algorithms compared.

3.2.1 Peak Picking

Peak picking is a simple algorithm to recognize a P3 component using the difference between the minimum and maximum amplitude in a trial. A trial with a prototypical evoked potential P3 component contains a large peak from 300-400ms and peak picking recognizes the P3 signal when the amplitude difference is greater than or equal to a specified voltage difference between the minimum and maximum voltage points within a specified time window as expressed by:

$$\max(x) - \min(x) \geq \tau \begin{cases} 1: \text{P3 Present} \\ 0: \text{P3 Absent} \end{cases} \quad (3)$$

where x is a vector which represents the data for a single evoked potential (EP) response and τ represents the threshold voltage difference required to accept a P3 signal. For recognition, the time window with the best results was between three and six hundred milliseconds. The voltage difference threshold was optimized for each subject using data from the Monitor condition.

3.2.2 Correlation

Correlation may be looked at as template matching when correlation is performed between single trials and templates of the P3 and non-P3 signals. Evoked potential responses were correlated with P3 and non-P3 component averages created from the Calibration task using the following formula:

$$\rho_{x,y} = \frac{\text{COV}(x, y)}{\sigma_x \sigma_y} \quad (4)$$

Where x is a vector which represents the data for an evoked potential response, y is a vector which represents the P3 or non-P3 signal average to compare against, $\text{COV}(x,y)$ is the covariance of x and y , and σ is the standard deviation of the appropriate signal.

To determine if a subject's response trial contains a P3 component, the correlation between an individual subject's response trial after triggering an item and the P3 signal average of that subject was compared with the evoked potential response trial and non-P3 signal average according to the algorithm:

$$\begin{aligned} &\text{if } \rho_{P3} > \tau \text{ and } \rho_{P3} \geq \rho_{\text{non-P3}} \text{ the P3 Present} \\ &\text{else P3 Absent} \end{aligned} \quad (5)$$

ρ_{P3} is the correlation between the EP response and P3 signal average, $\rho_{\text{non-P3}}$ is the correlation between the EP response and non-P3 signal average, and τ is the threshold set to determine the desired amount of correlation. The threshold was varied in experiments per subject. If the EP response did not correlate with a P3 or non-P3 signal average then the algorithm was indeterminate. During experimentation, all indeterminate responses were treated as not having P3 signals.

4 Results

Table 1 compares the best RV accuracies achieved by the three algorithms against the original accuracy. Peak picking and correlation achieved the same overall RV accuracy of 80%, however, correlation more significantly impacted the subjects' accuracies ($p < 0.008$) than peak picking ($p < 0.05$). All p values were derived using the paired Wilcoxon test.

In addition, Table 1 presents the theoretical best accuracy that can be achieved in this experiment when correcting mistakes. Overall, it was possible to improve the accuracy from 78% to 85%. This indicates that 15% of the errors were caused by missed goals, while 7% were caused by mistakes, and of those mistakes, 3% were corrected using variable averaging response verification.

Table 1: The change in accuracy when keeping subject responses classified with P3 signals using the peak picking and correlation algorithms for signal classification. The best theoretical accuracy occurs if all selected goals are kept while all selected non-goals are rejected (p calculated with a paired Wilcoxon test).

| Subject | Original Accuracy | New Accuracy | | Theoretical Best Accuracy |
|---------|-------------------|--------------|-------------|---------------------------|
| | | Peak Picking | Correlation | |
| 1 | 80 | 77 | 81 | 87 |
| 2 | 80 | 82 | 81 | 91 |
| 3 | 86 | 89 | 89 | 91 |
| 4 | 80 | 81 | 82 | 88 |
| 5 | 67 | 77 | 77 | 80 |
| 6 | 80 | 83 | 81 | 85 |
| 7 | 76 | 77 | 77 | 84 |
| 8 | 77 | 77 | 76 | 82 |
| 9 | 74 | 74 | 74 | 76 |
| Mean | 78 | 80 | 80 | 85 |
| Std | 5 | 5 | 4 | 5 |
| | | $p < 0.05$ | $p < 0.008$ | |

Table 2 shows the bit rate that subjects achieved when keeping an item triggered if a P3 component was present. The bits/trial equation used was from Pierce (1980) and is expressed as:

$$B_t = \log_2 N + P \log_2 P + (1 - P) \log_2 \frac{1 - P}{N - 1} \quad (6)$$

where there are N possible selections, each of which are equally probable, an accuracy probability of P . The bit rate B_m , equal to the bits/trial B_t , multiplied by the average number of trials/minute T_m :

$$B_m = B_t T_m \quad (7)$$

In the virtual apartment experiment N was 5 and T_m was 12 trials/minute. The overall increase in bit rate was 1.21 bits/minute and the greatest increase was 4.24 bits/minute by subject five. Subject five's change reflects the benefit of using automatic response error correction. From Table 1 we see subject five's original accuracy was 67% with 13% of their errors from mistakes. Through response error correction, the subject achieved 80% accuracy.

Table 2: Change in the bit rate from the original bit rate when keeping responses with P3 signals using the peak picking and correlation algorithms to recognize P3s. The best theoretical accuracy occurs if all selected goals are kept while all selected non-goals are rejected.

| Subject | Original Bit/minute | New Bits/minute | | Theoretical Bits/minute |
|---------|---------------------|-----------------|-------------|-------------------------|
| | | Peak Picking | Correlation | |
| 1 | 14.45 | 13.20 | 14.71 | 17.80 |
| 2 | 14.23 | 15.40 | 15.10 | 20.48 |
| 3 | 17.66 | 19.53 | 19.53 | 20.38 |
| 4 | 14.45 | 14.72 | 15.27 | 18.86 |
| 5 | 9.09 | 13.10 | 13.10 | 14.30 |
| 6 | 14.62 | 16.13 | 14.99 | 16.92 |
| 7 | 12.37 | 13.13 | 13.13 | 16.46 |
| 8 | 12.92 | 13.15 | 12.69 | 15.62 |
| 9 | 11.63 | 11.88 | 11.88 | 12.63 |
| Mean | 13.49 | 14.47 | 14.49 | 17.05 |
| Std | 2.38 | 2.32 | 2.25 | 2.65 |

5 Discussion and Future Work

The existence of the evoked potential P3 component for goal responses has been shown to aid in increasing the accuracy of a BCI system without decreasing the bit rate. The results appear robust and should be extended to a wider variety of algorithms to show possible system improvements. In addition, the technique needs to be used in an on-line experiment in order to show how error correction affects later subject responses. Since individuals adapt to BCI systems, off-line results often differ from those that are done on-line.

Automatic error correction for a brain-computer interface is useful and it is possible that the correction may be used in multiple types of BCI systems. While automatic error correction works for the discussed experimental paradigm, the existence of the evoked potential P3 component may be tied to the experimental paradigm used. Will other paradigms work and how easy is it to use these paradigms for other types of BCI systems? We hope that other BCI systems may consider using this signal in future work.

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